Indian Statistical Institute, Bangalore
B. Math (III)
Second Semester 2015-2016
Semester Examination : Statistics (IV)

Date: 27-04-2016 **Maximum Score 50** **Duration: 3 Hours**

1. Let $X_1, X_2, \cdots, X_N$ be a random sample from a continuous distribution with *cdf* F and *median* M. Let the distribution be symmetric about its median M. Explain Wilcoxon Signed Rank Test for testing the null hypothesis $H_0 : M = M_0$.

   [10]

2. Let $X_1, X_2, \cdots, X_m$ and $Y_1, Y_2, \cdots, Y_n$ be random samples from distributions with continuous *cdf* F_X and F_Y respectively, further the two random samples be independent of each other. Let $N = m + n$. Define $\boldsymbol{Z} = (Z_1, Z_2, \cdots, Z_N)$ for the combined ordered sample as $Z_i = 1(0)$ if the *ith* position is occupied by $X(Y)$ observation, $1 \leq i \leq N$; in the combined ordered sample. Under the null hypothesis $H_0 : F_X(t) = F_Y(t)$ for all $t \in \mathbb{R}$, prove that the distribution of $T_N(\boldsymbol{Z}) = \sum_{i=1}^{N} a_i Z_i$ is symmetric about its mean μ if N is even and the weights are $a_i = i$ for $i \leq \frac{N}{2}$ and $a_i = N - i + 1$ for $i > \frac{N}{2}$.

   [10]

3. Let $X_1, X_2, \cdots, X_m$ and $Y_1, Y_2, \cdots, Y_n$ be random samples from distributions with *cdf* F_X and F_Y respectively, further the two random samples be independent of each other. Obtain Mann Whitney U Test for testing the null hypothesis $H_0 : F_X(t) = F_Y(t)$ for all $t \in \mathbb{R}$. Show that the test is consistent.

   [15]

4. Explain how the logistic regression model is a member of the generalized linear models (GLM) family. Derive maximum likelihood estimators for the parameters of the logistic regression model and explain how to carry out likelihood ratio test for testing the hypothesis $H_0 : \beta = 0$. Explain the use of deviance for comparing model M say, with the saturated model for a *GLM*.

   [12]

5. Let $X_1, X_2, \cdots, X_m$ and $Y_1, Y_2, \cdots, Y_n$ be random samples from distributions with continuous *cdf* F_X and F_Y respectively, further the two random samples be independent of each other. Let $N = m+n$. Define $\boldsymbol{Z} = (Z_1, Z_2, \cdots, Z_N)$ for the combined ordered sample as $Z_i = 1(0)$ if the *ith* position is occupied by $X(Y)$ observation, $1 \leq i \leq N$; in the combined ordered sample. Explain Wilcoxon Rank Sum Test for testing the hypotheses $H_0 : F_X(t) = F_Y(t)$ for all $t \in \mathbb{R}$ versus $H_1 : F_Y(t) = F_X(t - \theta)$ for all $t \in \mathbb{R}$ and some $\theta \neq 0 \in \mathbb{R}$.

   [10]